Modals and conditionals

1 Limitations of the actual world[1]

Consider the fact that "Lewis Carroll" was the pen name of C.L. Dodgson. The following entailment seems valid:

(1) **A valid entailment:** (Winter, 2016: 194)

a. Lewis Carroll is Charles Lutwidge Dodgson.

b. Lewis Carroll wrote *Alice*.

c. $\Rightarrow$ Charles Lutwidge Dodgson wrote *Alice*.

What (1) highlights a *substitution property* of natural language: Lewis Carroll and C.L. Dodgson refer to the same individual, so we can substitute one for the other without changing truth conditions. This is predicted by the Principle of Compositionality.
This substitution property breaks down in certain contexts. Consider:

(2) **An invalid entailment:** (Winter, 2016: 192)

a. John believes [Lewis Carroll wrote *Alice*].

b. $\not\Rightarrow$ John believes [Charles Lutwidge Dodgson wrote *Alice*].

We expect the meanings of (2a) and (2b) to based on the meanings of (1a) and (1b), which should have the same truth values! They are both actually true!
Our current semantics is *extensional*: expressions denote their actual referents in the real world. An extensional semantics cannot model the data in (2).

(3) **Another puzzle:**

a. I hope that [I won the lottery].

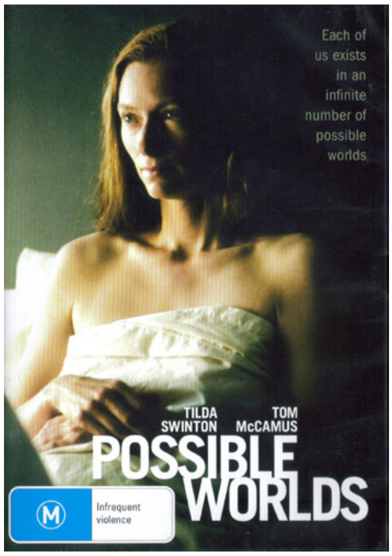
b. I hope that [a vaccine is found].

(4) a. I won the lottery. *false*

b. A vaccine is found. *false*

Intuition: Both of these puzzles above are problematic in our current semantics because *believe* and *hope* describe *how the world might be*, not just *how the world actually is*.
Therefore: We need to describe other worlds.

[1] This section's discussion follows Winter 2016.



"But things might have been different, in ever so many ways. This book of mine might have been finished on schedule... Or I might not have existed at all — neither myself, nor any counterpart of me. Or there might never have been any people... There are ever so many ways that a world might be: and one of these many ways is the way that this world is." David Lewis (1986)

(5) **Possible worlds:**

a. Worlds are type s

b. $W = D_s = \{w_1, w_2, w_3, ...\}$; w^* is the actual world

c. We enrich our denotation function with an *evaluation world* parameter: $\llbracket \cdot \rrbracket^w$

d. Names are fixed across worlds: for example, $\forall w, w' \in W \left[\llbracket \text{Tilda} \rrbracket^w = \llbracket \text{Tilda} \rrbracket^{w'} \right]$

e. Contradictions (like $2 + 2 = 5$) are false in all possible worlds.

f. Tautologies (like $1 + 1 = 2$) are true in all possible worlds.

Let's revisit the problematic examples above:

(6) **Beliefs in (2), revisited:**

a. $\llbracket \text{Lewis Carroll} \rrbracket^{w*} = \llbracket \text{C.L. Dodgson} \rrbracket^{w*}$ but there are other worlds where these descriptions do not give us the same referent.

b. $\llbracket \text{(2a)} \rrbracket = 1$ iff for all worlds w compatible with John's beliefs,
$\left[\llbracket \text{Lewis Carroll wrote } \textit{Alice} \rrbracket^w = 1 \right]$

c. $\llbracket \text{(2b)} \rrbracket = 1$ iff for all worlds w compatible with John's beliefs,
$\left[\llbracket \text{C.L. Dodgson wrote } \textit{Alice} \rrbracket^w = 1 \right]$

(7) **Hopes in (3), revisited:**

a. $\llbracket \text{(3a)} \rrbracket = 1$ iff for all worlds w where my hopes come true (or, ideal worlds),
$\left[\llbracket \text{I won the lottery} \rrbracket^w = 1 \right]$

b. $\llbracket \text{(3b)} \rrbracket = 1$ iff for all worlds w where my hopes come true (or, ideal worlds),
$\left[\llbracket \text{a vaccine is found} \rrbracket^w = 1 \right]$

Expressions that consider other possible worlds and therefore where the substitution property does not hold are called *intensional contexts*. (Not "intentional" with a *t*.)

2 Modals

Modals are a way to *quantify over (some) possible worlds*.

(8) ***Modal bases* = worlds to quantify over, a partial list:**

- *Epistemic*: worlds compatible with our knowledge
- *Deontic*: worlds that are compatible with laws and regulations
- *"Root"*: worlds compatible with the individuals' abilities

(9) ***Modal force* = the quantifier:**

- possibility: existential $\exists$ (traditionally $\Diamond$)
- necessity: universal $\forall$ (traditionally $\Box$)

(10) **"Weak" vs "strong" necessity:**

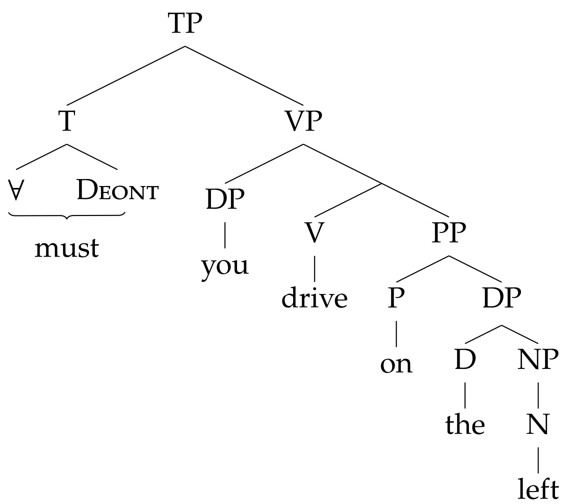
You **should** do the reading, but you don't **have to** ~~[$_{VP}$ do the reading]~~.

(11) **A modal base joke:**

a. Teacher: You **can**'t sleep in class.

b. Student: I know. You're talking too loud.

Intuition: Let's actually model modals as *the combination of a modal quantifier and a modal base*.[2]



(12) a. $[\![\text{EPIST}]\!] = \lambda w_s \,.\, w$ is compatible with the speaker's knowledge[3]

b. $[\![\text{DEONT}]\!] = \lambda w_s \,.\, w$ is compatible with relevant laws and regulations

(13) a. $[\![\forall]\!] = \lambda p_{\langle s,t\rangle} \,.\, \lambda q_{\langle s,t\rangle} \,.\, \forall w\ [p(w) \rightarrow q(w)]$

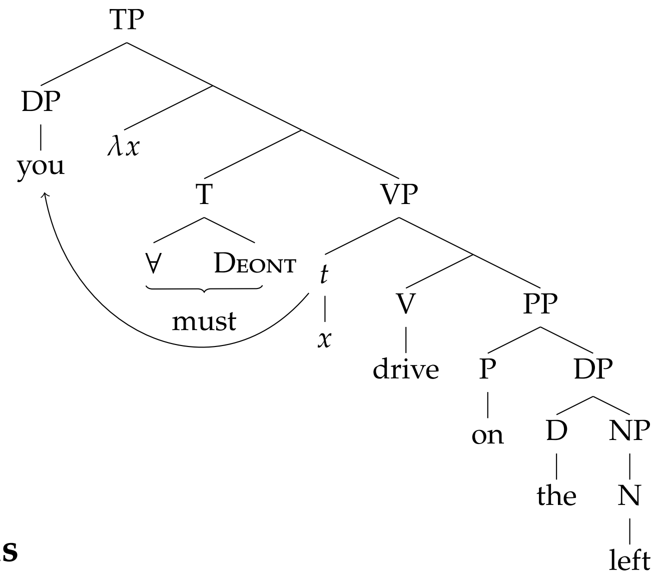
b. $[\![\exists]\!] = \lambda p_{\langle s,t\rangle} \,.\, \lambda q_{\langle s,t\rangle} \,.\, \exists w\ [p(w) \wedge q(w)]$

[2]This is a simplification, in many ways, from the state of the art; see von Fintel and Heim (2011).
[3]or sometimes other people's knowledge

We will generally continue to compute things extensionally — for example S/TP/VP will still generally be type t — although we carry the world variable w on the denotation function $[\![\cdot]\!]^w$. However, just when we need to, we will use a special rule that will turn a type t argument into its type $\langle s, t\rangle$ *intension*:

(14) **Intensional Functional Application:** (based on von Fintel and Heim, 2011)
If α is a branching node and $\{\beta, \gamma\}$ is the set of its daughters, then, for any world w and assignment g: if $[\![\beta]\!]^w$ is a function whose domain contains $\lambda w'_s \,.\, [\![\gamma]\!]^{w'}$, then $[\![\alpha]\!]^w = [\![\beta]\!]^w \left(\lambda w'_s \,.\, [\![\gamma]\!]^{w'}\right)$.

Again, in reality, the subject would move out:



3 Conditionals

3.1 Material implication

The classic analysis for "if p (then) q" is $p \to q$, which is equivalent to $(\neg p) \vee q$

(15) $[\![\text{if}]\!] = \lambda p_t \,.\, \lambda q_t \,.\, (\neg p) \vee q$

There are a number of problems with this view.

(16) From von Fintel and Heim 2011:

a. If there is a major earthquake in Cambridge tomorrow, my house will collapse. $p \to q$

b. It's not true that [if there is a major earthquake in Cambridge tomorrow, my house will collapse]. not $(p \to q)$

c. ≠ There will be a major earthquake in Cambridge tomorrow, and my house will fail to collapse. $p = 1$ and $q = 0$

Some additional problems with reasoning with conditionals as material implication:[4]

(17) Cantwell 2008: 331:

a. If you don't buy a lottery ticket, you can't win. (not p) $\rightarrow$ (not q)

b. You can win. q

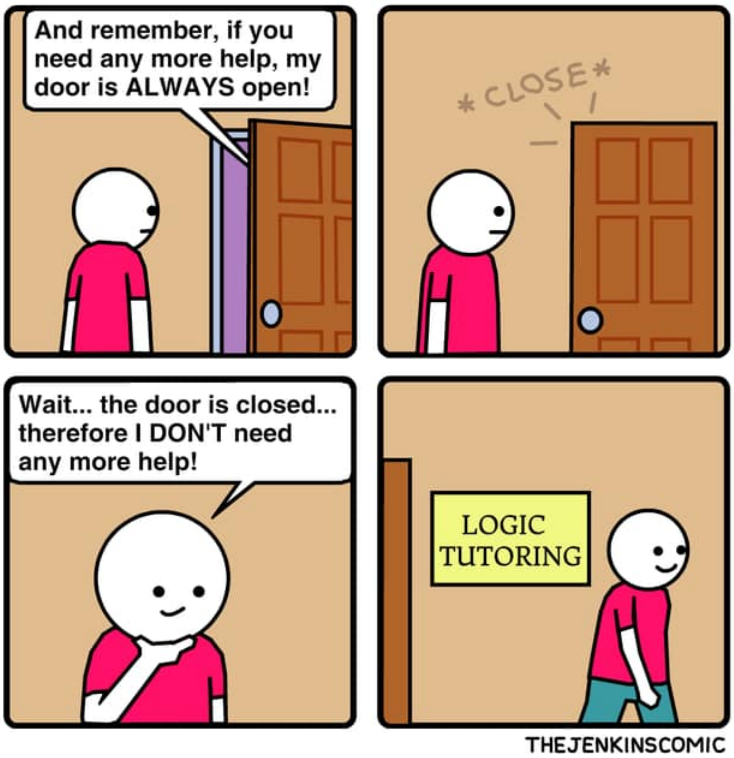
c. You do buy a lottery ticket. not(not p) $=$ p

(18) Yalcin 2012: 1003:

a. If there is a break-in, the alarm always sounds. $p \rightarrow q$

b. It is not the case that the alarm always sounds. not q

c. There is no break-in. not p



3.2 The modal restrictor view

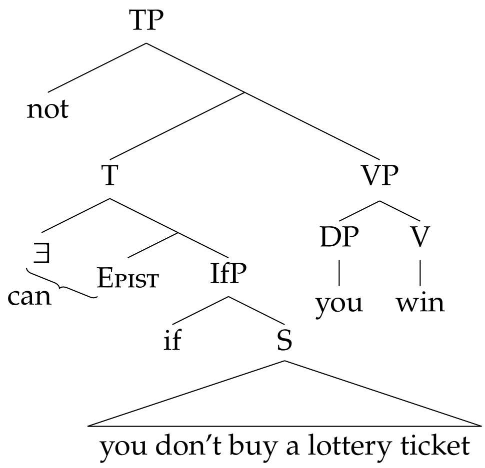
These paradoxes disappear if we think of the *if*-clause as *restricting the base* of a nearby modal.[5]

> "The history of the conditional is the story of a syntactic mistake. There is no two-place *if...then* connective in the logical forms for natural languages. *If*-clauses are devices for restricting the domains of operators." Kratzer (1986)

[4]These examples come from a collection of apparent counterexamples in the philosophical literature, compiled by Theresa Helke.

[5]The discussion in this section is based on discussions with Theresa Helke.

LF for (17a), pretending everything has reconstructed:

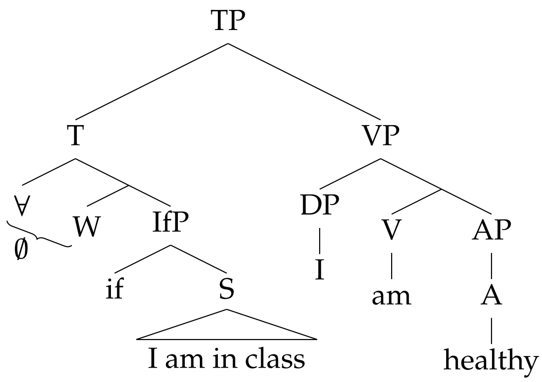


(19) $[\![\text{if}]\!] = \lambda p_{\langle s,t\rangle} \,.\, \lambda q_{\langle s,t\rangle} \,.\, \lambda w_s \,.\, p(w) \wedge q(w)$

Then what about conditionals without modals? Kratzer (1986) continues: "Bare indicative conditionals have unpronounced modal operators." Specifically, covert universal(-like) modals.

(20) If I am in class, I am healthy.

LF for (20), ignoring subject movement and the position of the conditional:



...where W is the $\langle s,t\rangle$ predicate true of all worlds, W = λw_s . 1 (the characteristic function of the set of all worlds)

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