Quantification

1 Quantifiers

The DPs we have studied so far have generally been of type e. Let's now consider subject DPs like *everyone*, *no one*,[1] and *someone*.

(1) Everyone sleeps.

Option 1: Include "plurals" in D_e, including a symbol that refers to 'nothing,' ϵ. *Everyone* is type e, the sum of all individuals.

(2) a. $D_e = \left\{ \begin{array}{c} \epsilon\text{, Alex, Brie, Cara,} \\ \text{Alex + Brie, Alex + Cara, Brie + Cara,} \\ \text{Alex + Brie + Cara} \end{array} \right\}$

b. $[\![\text{everyone}]\!] = \text{Alex + Brie + Cara (type } e)$

c. $[\![\text{everyone sleeps}]\!] = \text{Sleep(Alex + Brie + Cara)}$

This sort of works for *everyone*, but it does not work for *no one* and *someone*. Why?

Option 2: *Everyone* is not type e.

(3) a. $[\![\text{everyone}]\!] = \lambda Q_{\langle e,t\rangle} \,.\, \forall x \,[\text{Animate}(x) \rightarrow Q(x)]$

b. $[\![\text{everyone sleeps}]\!] = \forall x \,[\text{Animate}(x) \rightarrow \text{Sleep}(x)]$

Quantificational DPs are type $\langle\langle e,t\rangle,t\rangle$. In other words, they take the VP as their argument.

Exercise

(4) **Every** dog sleeps.

[1] Although we spell this as two words, "no one," we will treat it as one word, just like *nothing*.

2 Determiner meanings

We previously wrote meanings for quantificational determiners as relations between sets:

(5) **Quantificational determiners as set-relations:**

a. *every/all*$(A)(B) = 1$ iff $A \subseteq B$

b. *a/some*$(A)(B) = 1$ iff $A \cap B \neq \emptyset$

c. *no*$(A)(B) = 1$ iff $A \cap B = \emptyset$

d. *two*$(A)(B) = 1$ iff $\mid A \cap B \mid \geq 2$

e. *more-than-two*$(A)(B) = 1$ iff $\mid A \cap B \mid > 2$

f. *most*$(A)(B) = 1$ iff $\mid A \cap B \mid > \mid A \setminus B \mid$

Because we normally work with truth conditions and functions, not sets, we have to translate (5a) into non-set terms:

(6) $[\![\text{every dog sleeps}]\!] = 1 \text{ iff } \{x : \text{Dog}(x)\} \subseteq \{y : \text{Sleep}(y)\}$

$$\Leftrightarrow \forall z_e \in \{x : \text{Dog}(x)\} \ \big[z \in \{y : \text{Sleep}(y)\}\big]$$

$$\Leftrightarrow \forall z_e \ [\underbrace{\text{Dog}(z)}_{\textit{every}\text{'s first argument}} \rightarrow \underbrace{\text{Sleep}(z)}_{\textit{every}\text{'s second argument}}]$$

(7) $[\![\text{every}]\!] = \lambda P_{\langle e,t \rangle} \, . \, \lambda Q_{\langle e,t \rangle} \, . \, \forall z_e \ [P(z) \rightarrow Q(z)]$

We can similarly rewrite other quantificational determiners as λ functions of type $\langle \langle e,t \rangle, \langle \langle e,t \rangle, t \rangle \rangle$.

(8) $[\![\text{some}]\!] =$

(9) $[\![\text{no}]\!] =$

(10) $[\![\text{two}]\!] =$

For *every*, *a/some*, and *no*, we can write the determiner meanings using predicate logic. For others where we have to refer to the size (cardinality) of sets, we will still have to make reference to sets.

3 The definite determiner and presupposition calculation

(11) The black cat is in Texas.

A first approximation:

(12) $[\![\text{the}_{\text{sg}}]\!] = \lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.$ there is a unique x $[P(x) \wedge Q(x)]$ [2]

What meaning do we predict for (11)? Is that what (11) means?

(13) a. The elevator in AS5 is broken.

b. The escalator in AS5 is broken.

(14) **A "partial" semantics for the definite determiner:**[3]

$[\![\text{the}_{\text{sg}}]\!] = \lambda f : f \in D_{\langle e,t\rangle}$ and there is exactly one x such that $f(x) = 1$.
the unique y such that $f(y) = 1$

(15) $[\![\text{the black cat}]\!]$ = the unique black cat
$\rightsquigarrow$ there exists exactly one black cat (presupposition)

(16) **Functional Application (revised; compare to H&K p. 76):**[4]

If α is a branching node, $\{\beta, \gamma\}$ is the set of α's daughters, then

- $[\![\alpha]\!]$ is defined if and only if: $[\![\beta]\!]$ and $[\![\gamma]\!]$ are both defined and $[\![\beta]\!]$ is a function whose domain contains $[\![\gamma]\!]$;
- if defined, $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

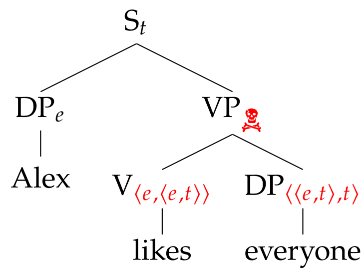
[2]This is not written in valid predicate logic, but we won't dwell on it as we won't adopt this meaning for *the* anyway. If we wanted to, we could write: $\lambda P_{\langle e,t\rangle} \,.\, \lambda Q_{\langle e,t\rangle} \,.\, \exists x[P(x) \wedge Q(x) \wedge \neg\exists y[y \neq x \wedge P(y) \wedge Q(y)]]$.

[3]A *partial function* is a function that is not defined for all possible values of its arguments.

[4]H&K describes this in terms of linguistic objects *being in the domain of* $[\![\cdot]\!]$ rather than being defined or not.

4 Quantifiers in object position

(17) Alex likes everyone.



We'll first consider the related sentence in (18), and then return to (17).

(18) Everyone, Alex likes ____.

Example (18) involves *movement* (specifically, topicalization) of the object. We need a semantics for how we interpret movement:

(19) **The interpretation of movement:**
Pick an arbitrary variable, such as x.

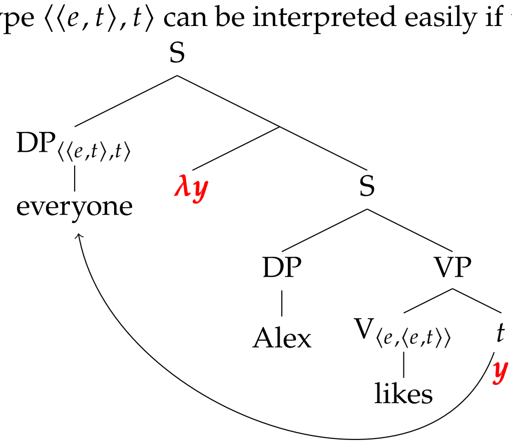
a. The base position of movement is replaced with a *trace*; $[\![t]\!] = x$, type e.

b. A *λ-binder* λx is adjoined right under the target position of the movement chain.

(20) **How to interpret λs in trees (aka λ Rule):** (to be revisited later)

$$\left[\!\!\left[\;[\; \lambda x \quad \ldots x \ldots \;]\; \right]\!\!\right] = \lambda x \,.\, \ldots x \ldots$$

Now notice that objects of type $\langle\langle e,t\rangle,t\rangle$ can be interpreted easily if they are moved:



Let's now return to example (17). One possible solution to the problem of quantifiers in object position is to interpret the sentence *as if the object has moved*, as in (18). The structure that we interpret is called *Logical Form* (LF).

(21) a. Surface structure: Alex likes everyone. =(17)

b. Logical Form (LF): everyone, Alex likes ____.

Such movement that is not reflected in the surface structure are called *covert*; I use dashed arrows for covert movement. The covert movement of quantifiers as in (21) is called *Quantifier Raising* (QR) (May, 1977). QR is required for quantifiers that are not in subject position, in order to avoid the type problem in (17).

References

May, Robert Carlen. 1977. The grammar of quantification. Doctoral Dissertation, Massachusetts Institute of Technology.