Plurality

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1 Introduction

Goals:

1. Introduce the notion of plurals
2. Show why we need a special approach to plurals
3. Distributive and collective predicates
4. Mereology
5. Open issues

2 Plurals

Familiar basic predications are easy to express in predicate logic.

(1) a. John walked (Walk(j))
    b. John is a student (Student(j))

But this isn't all there is to language! We also have expressions that refer to more than one individual.

(2) a. John and Mary walked
    b. John and Mary are students

How do we express these logically?

(3) A first attempt?
    a. John and Mary walked = Walk(j) $\wedge$ Walk(m)
    b. John walked and Mary walked

This doesn't work in every case though. Some predicates require a plural subject.

(4) a. John and Mary gathered.

b. *John gathered and Mary gathered

This shows us two things:

- Some predicates are semantically sensitive to plurality.
- It's not possible to translate plural expressions entirely as conjunctions of singulars.

3 Distributive and collective predicates

We notice that in some cases it is possible to treat predications of a plural entity as a conjunction of predications of singular entities, but not in others.

- The first kind of predicates, like *walk*, are called **distributive predicates**: if 'John and Mary walked' is true, 'John walked' is true, and so it 'Mary walked.'
- The second class of predicates, like *gather*, are called **collective predicates**: these only hold of plural entities, and they do not possess the same distributive property: 'John and Mary gathered in the park' does not mean that 'John gathered in the park.'

Some predicates are ambiguous between a collective and a distributive reading.

(5) John and Mary lifted the table
   a. Distributive: John lifted the table and put it down. Then Mary lifted the table. They *each* lifted the table.
   b. Collective: John and Mary lifted the table *together*.

Mereology

Mereology: a formal approach to representing *parthood*

- Applied to plurals, plurals are those expressions that have proper parts.
- This will be what we add to predicate logic to analyze plurals.

Singular individuals like *John* and *the student* are called **atoms** or **atomic individuals**: things that have no proper parts.

- They are represented as individual constants, as usual.

(6) $[\![\text{John}]\!] = \text{j}$

Plural individuals like *John and Mary* and *the students*, as represented as **sums** or **sum individuals**: they have proper parts.

- For any two individuals x and y, we write their sum as x⊕y.
- You can think of the ⊕ symbol as meaning 'and'

(7) ⟦John and Mary⟧ = j⊕m

In order to talk about how John and Mary are part of their sum j⊕m, we use the **parthood relation**, ≤.

(8) x ≤ y iff for some z, x⊕z = y

This means that x is part of y iff y is equal to the sum of x with something else. So j ≤ j⊕m, and m ≤ j⊕m.

We will treat singular predicates like *student* as denoting sets of atomic individuals.

(9) ⟦student⟧ = {j, m, b}

In order to make plural predicates out of singular ones, we will use a special operation, called the **algebraic closure operatior**.

- If P is a predicate, *P is its algebraic closure.
- * is often simply called the **star operator**

The star operator takes a set of things, and makes a new set with every possible way of making sums out of the things in the set.

(10) *P is the smallest set where
1) If a ∈ P, then a ∈ *P
2) If a and b are in *P, then a⊕b are in *P

(11) ⟦students⟧ = *⟦student⟧ = {j, m, b, j⊕m, j⊕b, m⊕b, j⊕m⊕b}

4 Plurals in predicate logic

We now have a way to write sentences with plurals in predicate logic.

(12) John and Mary walked = Walk(j⊕m)

(13) John and Mary are students = *Student(j⊕m)

Predicates like *walk* are distributive. We can represent this by saying if a sum is in the denotation of *walk*, then so are the atoms that are part of that sum.

(14) ⟦walk⟧ = {j⊕m, j⊕b, j, m, b, ...}

Collective predicates, on the other hand, are predicates that only have sums in their extensions. *Gather*, then, does not have any atomic individuals in its extension.

(15) $[\![\text{gather}]\!] = \{j\oplus m, j\oplus b, ...\}$

This is why *walk*, but not *gather*, can be paraphrased with conjunctions.

How do we deal with ambiguous predicates, like *lift a table*? One thing that we can do is treat the collective reading as basic. So *lift a piano* is collective by default, and can be turned into a distributive predicate.

- In order to do this, we use a **distributive operator**.

(16) $\text{Dist(P)} = \lambda x.\forall y[\text{Atom}(y) \wedge y \leq x \rightarrow P(y)]$

(17) John and Mary lifted a table
   a. Collective: $\text{LiftATable}(j\oplus m)$
   b. Distributive: $\text{Dist(LiftATable)}(j\oplus m) = \forall y[\text{Atom}(y) \wedge y \leq j\oplus m \rightarrow \text{LiftATable}(y)]$

5 Things to think about

1. Consider the translations of *student* and *students* again.

(18) $[\![\text{student}]\!] = \{j, m, b\}$

(19) $[\![\text{students}]\!] = {}^*[\![\text{student}]\!] = \{j, m, b, j\oplus m, j\oplus b, m\oplus b, j\oplus m\oplus b\}$

What sort of prediction do we make about a sentence like *John is students*?

2. We claimed that some predicates, like *gather*, are only acceptable with plural nouns. Is this true? Consider the following:

(20) John gathered the water

What is special about *water* that makes it okay with a collective predicate?

3. How might we put the pieces presented above together compositionally?