Problem Set 2

Due August 28 before class. Submit on Luminus > Files > Student Submission > PS2.

1. **Set notation practice:** *IFS* page 52, exercise 7

2. **Conservativity:** A quantifier Q is called *conservative* if $Q(A)(B)$ is true if and only if $Q(A)(A \cap B)$ is true. For example, the sentence "Every $\underline{\text{cat}}_A$ $\underline{\text{is hungry}}_B$" is true if and only if "Every $\underline{\text{cat}}_A$ $\underline{\text{is hungry and is a cat}}_{=(A \cap B)}$."

   (a) Using the definitions for these quantifiers from class, prove that *a*, *no*, and *more than two* are conservative.

   Example: We can show that *every* is conservative. We want to show that, for any sets A and B, *every*$(A)(B)$ is true if and only if *every*$(A)(A \cap B)$ is true. We have to show entailment in both directions.

   We first show that if *every*$(A)(B)$ is true, *every*$(A)(A \cap B)$ is true.

   Suppose *every*$(A)(B)$ is true, so $A \subseteq B$.

   Suppose $x \in A$.

   Because $A \subseteq B$, $x \in B$.

   Because $x \in A$ and $x \in B$, by definition of $\cap$, $x \in A \cap B$.

   Therefore, $A \subseteq A \cap B$. In other words, *every*$(A)(A \cap B)$ must be true.

   We next show that if *every*$(A)(A \cap B)$ is true, *every*$(A)(B)$ is true.

   Suppose *every*$(A)(A \cap B)$ is true, so $A \subseteq A \cap B$.

   Suppose $x \in A$.

   Because $A \subseteq A \cap B$, $x \in A \cap B$.

   Because $x \in A \cap B$, by definition of $\cap$, $x \in A$ and $x \in B$.

   Therefore, $A \subseteq B$. In other words, *every*$(A)(B)$ must be true.

   (b) Imagine the quantifier *allnon*, defined as follows: *allnon*$(A)(B)$ is true iff $D_e \setminus A \subseteq B$. (This is called *allnon* because it requires that "All non-A individuals are B.") Show that *allnon* is not conservative.

   Note: To show that Q is not conservative, it suffices to construct a model where $Q(A)(B)$ is true but $Q(A)(A \cap B)$ is false, or vice versa.