Problem Set 2

Due September 8 before class. Submit on IVLE > Files > Student Submission > PS2.

1. **Set notation exercise** from Heim & Kratzer pages 9–10:

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   The same set can be described in many different ways, often quite different superficially. Here you are supposed to figure out which of the following equalities hold and which ones don't. Sometimes the right answer is not just plain "yes" or "no", but something like "yes, but only if...". For example, the two sets in (i) are equal only in the special case where $a = b$. In case of doubt, the best way to check whether two sets are equal is to consider an arbitrary individual, say John, and to ask if John could be in one of the sets without being in the other as well.

   (a) $\{a\} = \{b\}$

   (b) $\{x : x = a\} = \{a\}$

   (c) $\{x : x \text{ is green}\} = \{y : y \text{ is green}\}$

   (d) $\{x : x \text{ likes } a\} = \{y : y \text{ likes } b\}$

   (e) $\{x : x \in A\} = A$

2. **Conservativity:** A quantifier Q is called *conservative* if $Q(A)(B)$ is true if and only if $Q(A)(A \cap B)$ is true. For example, the sentence "Every $\underline{\text{cat}}_A$ $\underline{\text{is hungry}}_B$" is true if and only if "Every $\underline{\text{cat}}_A$ $\underline{\text{is hungry and is a cat}}_{=(A \cap B)}$."

   (a) Using the definitions for these quantifiers from class, prove that *a*, *no*, and *more than two* are conservative.

   Example: We can show that *every* is conservative. We want to show that, for any sets A and B, $every(A)(B)$ is true if and only if $every(A)(A \cap B)$ is true. We have to show entailment in both directions.

   We first show that if $every(A)(B)$ is true, $every(A)(A \cap B)$ is true.
   Suppose $every(A)(B)$ is true, so $A \subseteq B$.
   Suppose $x \in A$.
   Because $A \subseteq B$, $x \in B$.
   Because $x \in A$ and $x \in B$, by definition of $\cap$, $x \in A \cap B$.
   Therefore, $A \subseteq A \cap B$. In other words, $every(A)(A \cap B)$ must be true.

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   We next show that if $every(A)(A \cap B)$ is true, $every(A)(B)$ is true.
   Suppose $every(A)(A \cap B)$ is true, so $A \subseteq A \cap B$.
   Suppose $x \in A$.
   Because $A \subseteq A \cap B$, $x \in A \cap B$.
   Because $x \in A \cap B$, by definition of $\cap$, $x \in A$ and $x \in B$.
   Therefore, $A \subseteq B$. In other words, $every(A)(B)$ must be true.

(b) Imagine the quantifier *allnon*, defined as follows: *allnon*$(A)(B)$ is true iff $D_e \setminus A \subseteq B$. (This is called *allnon* because it requires that "All non-A individuals are B.") Show that *allnon* is not conservative.

(c) Consider *only* as in sentences like "Only dogs bark." Define the truth conditions of *only*$(A)(B)$ in terms of the sets A and B. Show that this *only* is not conservative.

Note: To show that Q is not conservative, it suffices to construct a model where $Q(A)(B)$ is true but $Q(A)(A \cap B)$ is false, or vice versa.